

(1)

Exercice (1):

$$1) 6x^2 + 9x - 6 = 0 \quad \begin{cases} a = 6 \\ b = 9 \\ c = -6 \end{cases}$$

$$\Delta = b^2 - 4ac$$

$$= 9^2 - 4 \times 6 \times (-6)$$

$$= 81 + 144$$

$$= 225 > 0 \text{ l'équation admet}$$

2 solutions distinctes :

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-9 + 15}{12} = \frac{6}{12} = \frac{1}{2}$$

$$\text{et } x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-9 - 15}{12} = \frac{-24}{12} = -2$$

$$\text{Donc } S = \left\{ \frac{1}{2}; -2 \right\}$$

$$4) 3y^2 - y + 8 = 0 \quad \begin{cases} a = 3 \\ b = -1 \\ c = 8 \end{cases}$$

$$\Delta = b^2 - 4ac$$

$$= (-1)^2 - 4 \times 3 \times 8$$

$$= 1 - 96 = -95 < 0 \text{ Pas de solution réelle}$$

$$\text{Donc } S = \emptyset$$

$$2) 49 - (5x - 1)^2 = 0$$

$$\Leftrightarrow 7^2 - (5x - 1)^2 = 0$$

$$\Leftrightarrow (7 - (5x - 1))(7 + (5x - 1)) = 0$$

$$\Leftrightarrow (-5x + 8)(5x + 6) = 0$$

$$\Leftrightarrow -5x + 8 = 0 \text{ ou } 5x + 6 = 0$$

$$\Leftrightarrow x = \frac{8}{5} \text{ ou } x = -\frac{6}{5}$$

$$S = \left\{ \frac{8}{5}; -\frac{6}{5} \right\}$$

$$3) 64x^2 + 16x + 1 = 0$$

$$\Leftrightarrow (8x + 1)^2 = 0$$

$$\Leftrightarrow 8x + 1 = 0$$

$$\Leftrightarrow x = -\frac{1}{8}$$

$$5) \frac{t+1}{t+5} = \frac{4t-2}{t+17}$$

Valeurs interdites: -5 et -17

$$(t+1)(t+17) = (t+5)(4t-2)$$

$$\Leftrightarrow t^2 + 18t + 17 = 4t^2 + 18t - 10$$

$$\Leftrightarrow -3t^2 + 27 = 0$$

$$\Leftrightarrow t^2 = \frac{27}{3} = 9$$

$$\Leftrightarrow t = 3 \text{ ou } -3$$

$$S = \{-3; 3\}$$

Exercice (2):Par lecture graphique, $S(-2; 3)$

$$\text{Forme canonique de } f: f(x) = a(x - \alpha)^2 + \beta = a(x + 2)^2 + 3$$

$$\text{Or, } f(0) = 1 \text{ d'où: } a \times 4 + 3 = 1 \Leftrightarrow 4a = -2$$

$$\Leftrightarrow a = -\frac{2}{4} = -\frac{1}{2}$$

$$\text{Donc: } f(x) = -\frac{1}{2}(x+2)^2 + 3$$

Par conséquent, la forme développée et réduite de f :

(2)

$$f(x) = -\frac{1}{2}(x^2 + 4x + 4) + 3 = -\frac{1}{2}x^2 - 2x + 1$$

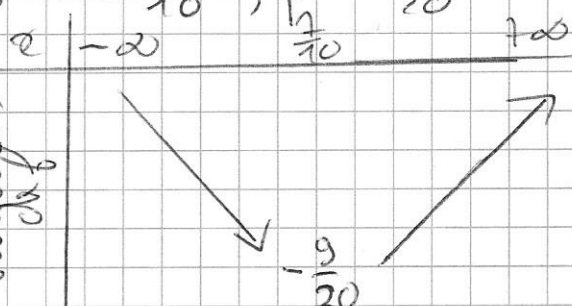
Exercice (3)

1) a) $f(x) = 5x^2 - 7x + 2$

$$\begin{aligned} &= 5\left(x^2 - \frac{7}{5}x + \frac{2}{5}\right) \\ &= 5\left(\left(x - \frac{7}{10}\right)^2 - \frac{49}{100} + \frac{2}{5}\right) \\ &= 5\left(\left(x - \frac{7}{10}\right)^2 - \frac{49}{100} + \frac{40}{100}\right) \\ &= 5\left(\left(x - \frac{7}{10}\right)^2 - \frac{9}{100}\right) \\ &= 5\left(x - \frac{7}{10}\right)^2 - \frac{9}{20} \end{aligned}$$

$a = 5 > 0$: f est d'abord décroissante, puis croissante

$$\alpha = \frac{7}{10}, \quad \beta = -\frac{9}{20}$$

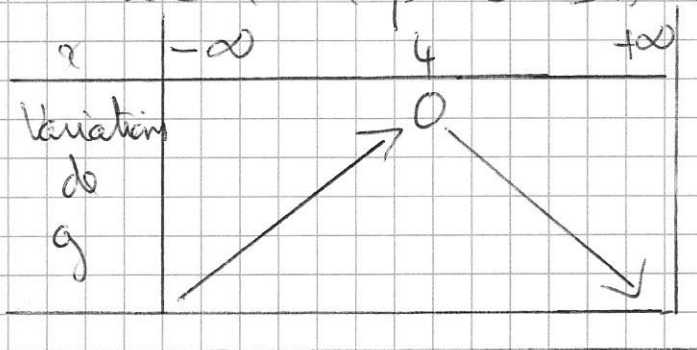


b) $g(x) = -3(x-4)^2$

$$= -3(x-4)^2 + 0 \quad (\text{forme canonique de } g)$$

$a = -3 < 0$, g est d'abord croissante, puis décroissante

$x = 4$ et $\beta = 0$ - On a le tableau de variations de g :



c) $R(x) = (x-1)^2 - (3x+4)^2$

$$= x^2 - 2x + 1 - (9x^2 + 24x + 16)$$

$$= -8x^2 - 26x - 15$$

$$\alpha = -\frac{b}{2a} = -\frac{26}{-16} = -\frac{13}{8}$$

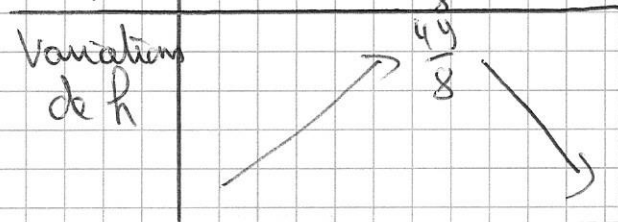
$$\beta = R(\alpha) = R\left(-\frac{13}{8}\right)$$

Forme canonique:

$$R\left(-\frac{13}{8}\right) = -8\left(-\frac{13}{8}\right)^2 - 26\left(-\frac{13}{8}\right) - 15$$

$$= -8 \times \frac{169}{64} + \frac{169}{4} - 15$$

$$= -\frac{169}{8} + \frac{338}{8} - \frac{120}{8} = \frac{169}{8} - \frac{120}{8} = \frac{49}{8}$$



Exercice (4):

3

$$f(x) = -x^2 - 8x + 20$$

1) Soit $x \in \mathbb{R}$:

$$\begin{aligned} f(x) &= -(x^2 + 8x - 20) \\ &= -((x+4)^2 - 16 - 20) \\ &= \underline{\underline{- (x+4)^2 + 36}} \end{aligned}$$

$$\begin{aligned} 2) f(x) &= 36 - (x+4)^2 \\ &= (6 + (x+4))(6 - (x+4)) \\ &= \underline{\underline{(x+10)(-x+2)}} \end{aligned}$$

$$\begin{aligned} 3) x \text{ est un antécédent de } 0 \text{ par } f &\Leftrightarrow f(x) = 0 \\ \text{Or, } f(x) = 0 &\Leftrightarrow (x+10)(-x+2) = 0 \\ &\Leftrightarrow x = -10 \text{ ou } x = 2 \end{aligned}$$

1^{ère} méthode

$$S = \underline{\underline{\{-10, 2\}}}$$

$$f(x) = 0 \Leftrightarrow -x^2 - 8x + 20 = 0 \quad \begin{cases} a = -1 \\ b = -8 \\ c = 20 \end{cases} \quad \text{2^{ème} méthode}$$

$$\begin{aligned} \Delta &= b^2 - 4ac = (-8)^2 - 4 \times (-1) \times 20 \\ &= 64 + 80 = 144 > 0 \text{ d'où l'équation admet deux} \\ \text{solutions : } x_1 &= \frac{-b + \sqrt{\Delta}}{2a} = \frac{8 + 12}{-2} = -10 \\ \text{et } x_2 &= \frac{-b - \sqrt{\Delta}}{2a} = \frac{8 - 12}{-2} = 2 \end{aligned}$$

Donc $S = \underline{\underline{\{-10, 2\}}}$

Exercice (5):

$$\begin{aligned} 1) (x+3)(x^2+x-30) &= x^3 + x^2 - 30x + 3x^2 + 3x - 90 \\ &= \underline{\underline{x^3 + 4x^2 - 27x - 90 = f(x)}} \end{aligned}$$

$$\begin{aligned} 2) f(x) = 0 &\Leftrightarrow (x+3)(x^2+x-30) = 0 \\ &\Leftrightarrow x+3 = 0 \text{ ou } x^2+x-30 = 0 \quad (\text{E}) \\ &\Leftrightarrow x = -3 \text{ ou } x^2+x-30 = 0. \end{aligned}$$

$$(E): x^2 + x - 30 = 0$$

$$\begin{cases} a = 1 \\ b = 1 \\ c = -30 \end{cases}$$

(4)

$$\Delta = b^2 - 4ac$$

$$= 1 - 4 \times 1 \times (-30)$$

$= 121 > 0$ d'où l'équation admet 2 solutions distinctes.

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-1 + 11}{2} = 5 \quad \text{et} \quad x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-1 - 11}{2} = -6$$

Par conséquent: $S = \{-3, 5, -6\}$

$$= -6$$

Exercice (6): $f(x) = ax^2 + bx + c \quad (a \neq 0)$

$$\Delta = b^2 - 4ac > 0$$

$$1) x_1 = \frac{-b + \sqrt{\Delta}}{2a} \quad \text{et} \quad x_2 = \frac{-b - \sqrt{\Delta}}{2a}$$

$$x_1 + x_2 = \frac{-b + \sqrt{\Delta} - b - \sqrt{\Delta}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$$

$$x_1 \times x_2 = \left(\frac{-b + \sqrt{\Delta}}{2a} \right) \times \left(\frac{-b - \sqrt{\Delta}}{2a} \right)$$

$$= \frac{1}{4a^2} ((-b)^2 - (\sqrt{\Delta})^2) = \frac{1}{4a^2} (b^2 - (b^2 - 4ac))$$

$$= \frac{1}{4a^2} (\cancel{b^2} - \cancel{b^2} + 4ac)$$

$$= \frac{4ac}{4a^2} = \frac{c}{a}$$

$$3) a) \text{ Soit } x_1 = -3$$

$$x_2 = -\frac{b}{a} - x_1 = -\frac{b}{a} + 3$$

b) Posons par exemple $a = 1$ et $b = 2$

$$\text{alors } -\frac{b}{a} = -\frac{2}{1} = -2 \quad \text{d'où } x_2 = -2 + 3 = 1$$

on peut avoir alors $f(x) = x^2 + 2x + c$

$$\text{or, } \frac{c}{a} = 1 \times (-3) = -3 \quad \Leftrightarrow c = -3a = -3$$

$$\text{Donc: } \underline{f(x) = x^2 + 2x - 3}$$